

Section 3A. Linear Algebra Review

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Basic Concepts and Notation

- ▶ We denote by $A \in \mathbb{R}^{m \times n}$ a *matrix* with m rows, n columns, where the entries are real numbers.
 - ▶ We typically use capital letters for matrices.
 - ▶ The element of A in the i -th row and j -th column is denoted by either $[A]_{ij}$ or A_{ij}
 - ▶ The *transpose* of a matrix A is denoted by $A^\top$, i.e., $[A^\top]_{ij} = [A]_{ji}$. In other words, the transpose results from converting rows of A into columns of $A^\top$. The transpose operation satisfies: $(A^\top)^\top = A$, $(AB)^\top = B^\top A^\top$, $(A + B)^\top = A^\top + B^\top$.

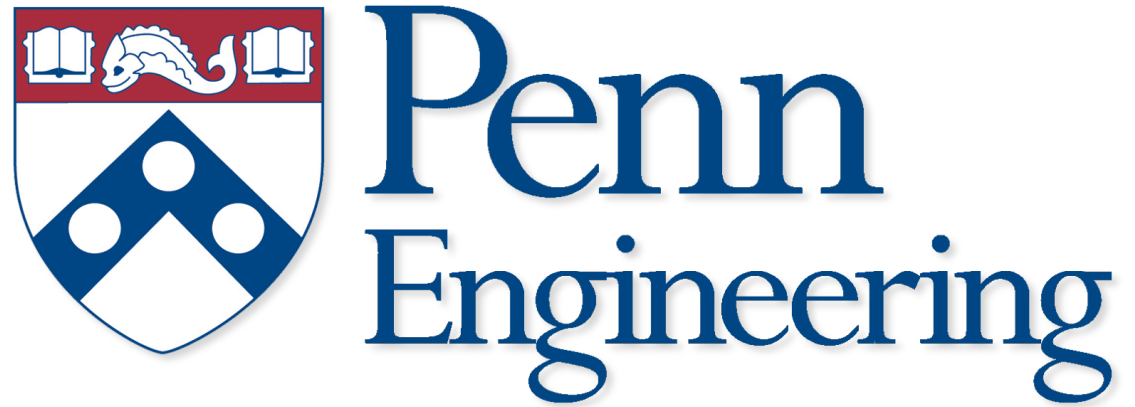
Matrix Algebra

- ▶ Given two matrices $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times p}$, the product $C = A \cdot B \in \mathbb{R}^{m \times p}$ is defined entry-wise as

$$C_{ij} = \sum_{k=1}^n A_{ik} B_{kj}$$

- ▶ Matrix multiplication is associative (i.e., $(AB)C = A(BC)$), distributive (i.e., $A(B + C) = AB + AC$), but (in general) not commutative (i.e., $AB \neq BA$)
- ▶ The identity matrix, denoted by $I_n \in \mathbb{R}^{n \times n}$, is a square matrix with ones on the diagonal and zeros everywhere else. This matrix satisfies

$$AI_n = A = I_n A$$



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